gr-qc/0703019

Transition from AdS universe to DS universe in the BPP model

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(Dated: March 6, 2007)

Abstract

It can be shown that in the BPP model the smooth phase transition from the asymptotically decelerated AdS universe to the asymptotically accelerated DS universe is possible by solving the modified semiclassical equations of motion. This transition comes from noncommutative Poisson algebra, which gives the constant curvature scalars asymptotically. The decelerated expansion of the early universe is due to the negative energy density with the negative pressure induced by quantum back reaction, and the accelerated late-time universe comes from the positive energy and the negative pressure which behave like dark energy source in recent cosmological models.

PACS numbers: 02.40.Gh, 04.60.-m, 98.80.Qc

Keywords: 2D Gravity, Models of Quantum Gravity, Non-CommutativeGeometry, Cosmology of Theories beyond the SM

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I. INTRODUCTION

One of the intriguing issues is not only to describe the late-time accelerated expansion of our universe but also to explain the smooth transition from decelerating phase to accelerating one. In the context of Einstein theory of general relativity, the accelerating universe means that the parameter $\omega \equiv p/\varepsilon$ of equation of state [1] is negative, where ε and p are the energy density and the pressure, respectively. So, in the ordinary Friedmann equation, the energy density is assumed to be positive while the pressure is negative. Even more $\omega < -1$ can be required to compensate the effect of ordinary matters in our universe. In some sense, it implies that the state parameter may depend on time and make it possible to explain the transition from decelerating phase to accelerating one. In the quintessence model based on supergravity or M/string theory, the transition has been studied in terms of the numerical simulation [2].

On the other hand, a two-dimensional dilaton gravity may be useful in studying the transition from the decelerated phase and the accelerated phase because there are fewer degrees of freedom rather than the four-dimensional counterpart. Furthermore, there exist exactly soluble models semiclassically [3, 4, 5, 6, 7, 8], whose quantum back reactions of the geometry are easily treated so that various cosmological problems have been studied in Refs. [7, 9, 10, 11, 12, 13]. However, in even this semiclassically soluble gravity, it is difficult to realize the smooth phase transition because the solution shows the only decelerating or accelerating behavior. Recently, it has been shown that it is possible to obtain the transition from decelerating phase to accelerating one by assuming the modified Poisson brackets [14] corresponding to noncommutativity of fields [15, 16, 17]. Unfortunately, the future singularity appears at finite time in this model, and the decelerated geometry has been patched by hand for the regularity.

So, in this paper, we would like to study the smooth phase transition from the decelerated expansion to accelerated expansion without any curvature singularity in the Bose-Parker-Peleg (BPP) model [6], which is one of the exactly soluble model semiclassically. In particular, even though the classical cosmological constant is not assumed, the initial state is asymptotically anti-de Sitter (AdS) and the late time behavior of our universe is asymptotically de Sitter (DS). This interesting feature is due to the noncommutativity in the modified Poisson algebra. In Sec. II, we find the semiclassical Hamiltonian in the BPP model and also define semiclassical energy-momentum tensors, and obtain the energy density and the pressure in view of a perfect fluid. In Sec. III, solving the semiclassical Hamiltonian equations of motion with the ordinary Poisson brackets in the BPP model, we obtain the accelerated expansion solution. In Sec. IV, we will take the modified Poisson brackets instead of the conventional Poisson algebra. Under some conditions for integration constants, the solution shows that the smooth transition from AdS (decelerating) phase at the past infinity to DS (accelerating) phase is possible. Finally, some discussions are given in Sec. V.

II. HAMILTONIAN AND ENERGY-MOMENTUM TENSORS

In the low-energy string theory, the two-dimensional dilaton gravity are described by

$$S_{\rm DG} = \frac{1}{2\pi}\int d^2x\sqrt{-g}e^{-2\phi}\left[R + 4(\nabla\phi)^2 + 4\lambda^2\right], \tag{1}$$

and the conformal matter fields is given as

$$S_{\rm cl} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[-\frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right], \tag{2}$$

where ϕ and f_i's are the dilaton and the conformal matter fields, respectively. We set the vanishing cosmological constant $\lambda^2 = 0$ for simplicity in what follows. The quantum effective action for the conformal matter (2) is written as

$$S_{\rm qt} = \frac{\kappa}{2\pi}\int d^2x\sqrt{-g}\left[-\frac{1}{4}R\frac{1}{\Box}R + (\nabla\phi)^2 - \phi R\right], \tag{3}$$

where $\kappa = (N-24)/12$. The first term in Eq. (3) comes from the Polyakov effective action of the classical matter fields [3, 5] and the other two local terms have been introduced in order to solve the semiclassical equations of motion exactly [6]. The higher order of quantum correction beyond the one-loop is negligible in the large N approximation where $N \to \infty$ and $\hbar \to 0$, so that κ is assumed to be positive finite constant.

In order to study consider the quantum back reaction semiclassically, we take the total action as

$$S = S_{\rm DG} + S_{\rm cl} + S_{\rm qt}. \tag{4}$$

In the conformal gauge, $ds^2 = -e^{2\rho}dx^+dx^-$, the total action and the constraint equations are written as

$$S \;=\; \frac{1}{\pi}\int d^2x\Big[e^{-2\phi}\left(2\partial_+\partial_-\rho - 4\partial_+\phi\partial_-\phi\right) - \kappa(\partial_+\rho\partial_-\rho + 2\phi\partial_+\partial_-\rho$$

$$+\partial_+\phi\partial_-\phi) + \frac{1}{2}\sum_{i=1}^{N}\partial_+ f_i \partial_- f_i\Big] \tag{5}$$

and

$$e^{-2\phi}\left[4\partial_\pm\rho\partial_\pm\phi - 2\partial_\pm^2\phi\right] + \frac{1}{2}\sum_{i=1}^{N}(\partial_\pm f_i)^2 + \kappa\left[\partial_\pm^2\rho - (\partial_\pm\rho)^2\right]$$
$$-\kappa\left(\partial_\pm^2\phi - 2\partial_\pm\rho\partial_\pm\phi\right) - \kappa\left(\partial_\pm\phi\right)^2 - \kappa t_\pm = 0, \tag{6}$$

where $t_\pm$ reflects the nonlocality of the induced gravity of the conformal anomaly. Then, we take the vanishing classical matter, $f_i = 0$ in order to take into account only the quantum-mechanically induced source. Defining new fields as [4, 6]

$$\Omega = e^{-2\phi}, \tag{7}$$

$$\chi = \kappa(\rho - \phi) + e^{-2\phi}, \tag{8}$$

the gauge fixed action is obtained in the simplest form of

$$S = \frac{1}{\pi}\int d^2x\left[\frac{1}{\kappa}\partial_+\Omega\partial_-\Omega - \frac{1}{\kappa}\partial_+\chi\partial_-\chi\right] \tag{9}$$

and the constraints are given by

$$\kappa t_\pm = \frac{1}{\kappa}\left(\partial_\pm\Omega\right)^2 - \frac{1}{\kappa}\left(\partial_\pm\chi\right)^2 + \partial_\pm^2\chi. \tag{10}$$

In the homogeneous space, using the relations of $x^\pm = t \pm x$, the Lagrangian and the constraints are obtained,

$$L = \frac{1}{2\kappa}\dot{\Omega}^2 - \frac{1}{2\kappa}\dot{\chi}^2, \tag{11}$$

$$\frac{1}{4\kappa}\dot{\Omega}^2 - \frac{1}{4\kappa}\dot{\chi}^2 + \frac{1}{4}\ddot{\chi} - \kappa t_\pm = 0, \tag{12}$$

where the action is redefined by $S/L_0 = \frac{1}{\pi}\int dtL$ with $L_0 = \int dx$, and the overdot denotes the derivative with respect to the conformal time t. Then, the Hamiltonian becomes

$$H = \frac{\kappa}{2}P_\Omega^2 - \frac{\kappa}{2}P_\chi^2 \tag{13}$$

in terms of the canonical momenta $P_\chi = -\frac{1}{\kappa}\dot{\chi}$, $P_\Omega = \frac{1}{\kappa}\dot{\Omega}$.

Since the semiclassical energy-momentum tensors are defined by $T_{\mu\nu}^{\rm qt} \equiv -(2\pi/\sqrt{-g})(\delta S_{\rm qt}/\delta g^{\mu\nu})$, they can be written as

$$T_{\pm\pm}^{\rm qt} = -\kappa t_\pm + \kappa\partial_\pm^2(\chi - \Omega) - \kappa[\partial_\pm(\chi - \Omega)]^2$$

$$
\begin{aligned}
&= -\kappa t_{\pm} + \frac{1}{4}(\ddot{\chi} - \ddot{\Omega}) - \frac{1}{4\kappa}(\dot{\chi} - \dot{\Omega})^2, & (14)\\
T^{\rm qt}_{+-} &= -\kappa \partial_+ \partial_- (\chi - \Omega)\\
&= -\frac{1}{4}(\ddot{\chi} - \ddot{\Omega}). & (15)
\end{aligned}
$$

They can be regarded as a perfect fluid written in the form of

$$
T^{\rm qt}_{\mu\nu} = p g_{\mu\nu} + (p+\varepsilon) u_\mu u_\nu, \tag{16}
$$

where ε and p are the energy density and the pressure, respectively, and u^μ is the 4-velocity vector field of flow. In the comoving coordinate, $ds^2 = -d\tau^2 + a^2(\tau)dx^2$, the 4-velocity is given by $u_\mu = (1,0)$, and then we can obtain the distributions of the energy density and the pressure. Note that the comoving time are related to the conformal time, $\tau = \int e^\rho dt = \int \Omega^{-1/2} \exp[(1/\kappa)(\chi - \Omega)]dt$. Then, the energy density and pressure are written as

$$
\begin{aligned}
\varepsilon &= T^{\rm qt}_{\tau\tau} = e^{-2\rho}(T^{\rm qt}_{++} + 2T^{\rm qt}_{+-} + T^{\rm qt}_{--}), & (17)\\
p &= \frac{1}{a^2} T^{\rm qt}_{xx} = e^{-2\rho}(T^{\rm qt}_{++} - 2T^{\rm qt}_{+-} + T^{\rm qt}_{--}). & (18)
\end{aligned}
$$

Note that the state parameter ω has been defined as the equation of state $p = \omega\varepsilon$.

III. ACCELERATED UNIVERSE WITH CONVENTIONAL POISSON BRACKETS

In this section, we would like to recapitulate the evolution of the two-dimensional universe by solving the semiclassical equations of motion in the BPP model. Even if the solutions can be obtained directly from the Lagrangian equations of motion, we will solve them in terms of the Hamiltonian formulation since the latter case is more convenient to modify the original equations of motion. Let us now define the conventional Poisson brackets,

$$
\{\Omega, P_\Omega\}_{\rm PB} = \{\chi, P_\chi\}_{\rm PB} = 1, \quad \text{others} = 0 \tag{19}
$$

and then the Hamiltonian equations of motion in Ref. [18] are given by $\dot{\mathcal{O}} = \{\mathcal{O}, H\}_{\rm PB}$ where $\mathcal{O}$ represents fields and corresponding momenta. Then they are explicitly written as

$$
\dot{\chi} = -\kappa P_\chi, \quad \dot{\Omega} = \kappa P_\Omega, \tag{20}
$$

$$
\dot{P}_\chi = 0, \quad \dot{P}_\Omega = 0. \tag{21}
$$

Since the momenta P_Ω and P_χ are constants of motion, we can easily obtain the solutions,

$$\Omega = \kappa P_{\Omega_0} t + A_0, \tag{22}$$

$$\chi = -\kappa P_{\chi_0} t + B_0, \tag{23}$$

where $P_\Omega = P_{\Omega_0}$, $P_\chi = P_{\chi_0}$, A_0, and B_0 are arbitrary constants. From the definition (7), the solution Ω in Eq. (22) must be positive. This leads to three cases of conformal time t: one is $t > A_0/(\kappa P_{\Omega_0})$ with $P_{\Omega_0} > 0$, another is $t < A_0/(\kappa P_{\Omega_0})$ with $P_{\Omega_0} < 0$, and the other is $-\infty < t < \infty$ with $P_{\Omega_0} = 0$ and $A_0 > 0$.

Next, the dynamical solutions (22) and (23) should by satisfied with constraint (12), which results in

$$\kappa t_\pm = \frac{\kappa}{4}(P_{\Omega_0}^2 - P_{\chi_0}^2). \tag{24}$$

Note that the integration functions $t_\pm$ determined by the matter state are time-independent. On the other hand, by using Eqs. (22) and (23), the curvature scalar is calculated as

$$R = \frac{2}{a}\frac{d^2 a}{d\tau^2} = \kappa^2 P_{\Omega_0}^2 e^{-2\rho+4\phi} = \kappa^2 P_{\Omega_0}^2 \frac{e^{-2B_0+2\kappa P_{\chi_0} t}}{A_0 + \kappa P_{\Omega_0} t} \geq 0, \tag{25}$$

where the equality corresponds to the case of $P_{\Omega_0} = 0$ and $A_0 > 0$, in other words, which means flat spacetime.

Plugging the constraint (24) into Eqs. (14) and (15), the induced energy-momentum tensors are explicitly written as

$$T^{\rm qt}_{\pm\pm} = -\frac{\kappa}{2} P_{\Omega_0}(P_{\Omega_0} + P_{\chi_0}), \tag{26}$$

$$T^{\rm qt}_{+-} = 0, \tag{27}$$

which yields from Eqs. (17) and (18),

$$\varepsilon = p = -\kappa P_{\Omega_0}(P_{\Omega_0} + P_{\chi_0})(\kappa P_{\Omega_0} t + A_0) \exp\left[\frac{2}{\kappa}(A_0 - B_0) + 2(P_{\Omega_0} + P_{\chi_0})t\right]. \tag{28}$$

Note that the state parameter is simply $\omega = 1$ in this semiclassical case, and the curvature scalar which is proportional to the acceleration is always positive under the condition of $\Omega > 0$. So, there is no phase transition from the deceleration to the acceleration, and we can not obtain the AdS-DS phase transition.

IV. ADS-DS PHASE TRANSITION WITH MODIFIED POISSON BRACKETS

In this section, we now study whether the phase change of the universe is possible or not in the context of the modified semiclassical equations of motion. The similar analysis to the previous section will be done along with the noncommutative algebra [15, 19],

$$\{\Omega, P_\Omega\}_{\rm MPB} = \{\chi, P_\chi\}_{\rm MPB} = 1,$$
$$\{\chi, \Omega\}_{\rm MPB} = 0, \quad \{P_\chi, P_\Omega\}_{\rm MPB} = \theta, \quad \text{others} = 0, \tag{29}$$

where θ is a positive constant. Note that our starting semiclassical action seems to be quantized one more, however, this is not the case since these modified Poisson brackets are simply the counterpart of the conventional Poisson brackets which are not quantum commutators. If the fields had been taken as operators by decomposing the positive and the negative frequency modes along with the normal ordering, then it would be the quantization of a quantization. But our modified Poisson brackets just modify the conventional (semiclassical) Hamiltonian equations of motion, which still result in the semiclassical solutions, of course, they are θ-dependent due to the modification of the Poisson brackets.

Using the Hamiltonian (13), the previous equations of motion are promoted to the followings,

$$\dot{\chi} = \{\chi, H\}_{\rm MPB} = -\kappa P_\chi, \quad \dot{\Omega} = \{\Omega, H\}_{\rm MPB} = \kappa P_\Omega, \tag{30}$$
$$\dot{P}_\chi = \{P_\chi, H\}_{\rm MPB} = \kappa\theta P_\Omega, \quad \dot{P}_\Omega = \{P_\Omega, H\}_{\rm MPB} = \kappa\theta P_\chi. \tag{31}$$

Note that the momenta are no more constants of motion because of nonvanishing θ, hereby, a new set of equations of motion from Eqs. (30) and (31) are obtained,

$$\ddot{\chi} = -\kappa\theta\dot{\Omega}, \quad \ddot{\Omega} = -\kappa\theta\dot{\chi}. \tag{32}$$

Of course, the parameter θ is independent of the quantization where the modified semiclassical equations of motion (31) is reduced to Eq. (21) for $\theta \to 0$. From the coupled equations of motion (32), we obtained the solutions as

$$\Omega = e^{-2\phi} = \alpha e^{-\kappa\theta t} + \beta e^{\kappa\theta t} + A, \tag{33}$$
$$\chi = e^{-2\phi} + \kappa(\rho - \phi) = \alpha e^{-\kappa\theta t} - \beta e^{\kappa\theta t} + B, \tag{34}$$

where κ has been assumed to be a positive constant, and α, β, A, and B are constants of integration. Since Ω should be positive in Eq. (33), the constants α, β, and A are appropriately restricted. Then, the scale factor and the expanding velocity are given as

$$a(\tau) \ = \ e^{\rho(t)} = \frac{\exp[-\frac{1}{\kappa}(A-B) - \frac{2}{\kappa}\beta e^{\kappa\theta t}]}{\sqrt{\alpha e^{-\kappa\theta t} + \beta e^{\kappa\theta t} + A}}, \tag{35}$$

$$\frac{da}{d\tau} \ = \ \dot{\rho} = \frac{1}{2}\theta \frac{\kappa(\alpha e^{-\kappa\theta t} - \beta e^{\kappa\theta t}) - (2\beta e^{\kappa\theta t} + A)^2 + A^2 - 4\alpha\beta}{\alpha e^{-\kappa\theta t} + \beta e^{\kappa\theta t} + A}, \tag{36}$$

respectively, where we used $dt/d\tau = e^{-\rho(t)}$ and $a(\tau) = e^{\rho(t)}$. The overdot denotes the derivative with respect to t and comoving time τ is related to conformal time t by $\tau = \int e^{\rho(t)} dt$, which can be explicitly calculated from the scale factor (35). Subsequently, the acceleration and the curvature scalar are calculated as

$$\begin{aligned} \frac{d^2a}{d\tau^2} \ &= \ e^{-\rho}\ddot{\rho} = \frac{1}{2}\kappa\theta^2 \frac{\exp[\frac{1}{\kappa}(A-B) + \frac{2}{\kappa}\beta e^{\kappa\theta t}]}{\sqrt{\alpha e^{-\kappa\theta t} + \beta e^{\kappa\theta t} + A}} \Big[\kappa \frac{A^2 - 4\alpha\beta}{\alpha e^{-\kappa\theta t} + \beta e^{\kappa\theta t} + A} \\ &\quad -(2\beta e^{\kappa\theta t} + A)^2 - 4\alpha\beta + A^2 - \kappa A\Big], \end{aligned} \tag{37}$$

$$\begin{aligned} R \ &= \ \frac{2}{a}\frac{d^2a}{d\tau^2} = \kappa\theta^2 \exp\left[\frac{2}{\kappa}(A-B) + \frac{4}{\kappa}\beta e^{\kappa\theta t}\right] \Big[\kappa \frac{A^2 - 4\alpha\beta}{\alpha e^{-\kappa\theta t} + \beta e^{\kappa\theta t} + A} \\ &\quad -(2\beta e^{\kappa\theta t} + A)^2 - 4\alpha\beta + A^2 - \kappa A\Big], \end{aligned} \tag{38}$$

respectively.

In order to describe the smooth transition from the decelerated phase to the accelerated universe, eventually, from the AdS to the DS phase, we will consider the special case of $\alpha > 0$, $\beta > 0$, and $A < -\kappa$ with the condition $A^2 = 4\alpha\beta$ in what follows. These constants tells us that the range of the conformal time is $t < (\kappa\theta)^{-1}\ln(-A/2\beta)$ as seen from Eq. (33), and then the range of the comoving time should be $\tau > 0$. Under this restriction, the expanding velocity $da/d\tau$ is always positive and the scale factor increases from zero to infinity. Note that $\tau(t)$ is a monotonic increasing function with respect to t. On the other hand, the acceleration $d^2a/d\tau^2$ is zero at the initial time $\tau = 0$ and is negative before $\tau = \tau_1$, where $\tau_1 = \int_{-\infty}^{t_1} e^{\rho(t)} dt$ where $t_1 = (\kappa\theta)^{-1}\ln[(2\beta)^{-1}(-A - \sqrt{-\kappa A})]$. After $\tau = \tau_1$, the acceleration becomes positive, which shows the smooth phase transition. Although the acceleration diverges as τ goes to infinity, but there exists no curvature singularity as shown in Fig. 1 due to the infinite scale factor. In fact, the curvature scalar is almost negative constant, $R \approx -A(\kappa + A)\kappa\theta^2 \exp[\frac{2}{\kappa}(A-B)] < 0$, around $\tau = 0$ and it becomes zero at $\tau = \tau_1$, and then approaches the positive constant, $R \approx -A\kappa^2\theta^2 \exp(-\frac{2}{\kappa}B) > 0$, at $\tau \to \infty$. This fact shows that the phase transition from AdS universe to DS appears.

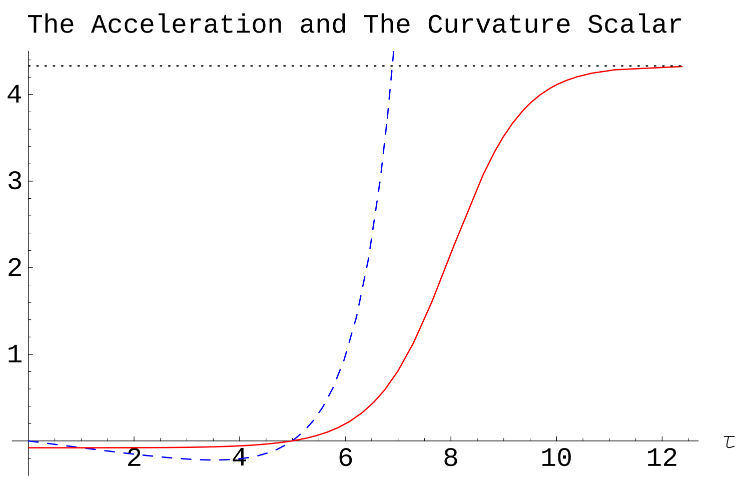

FIG. 1: The solid line and the dashed line denote the curvature scalar and the acceleration of the scale factor, respectively. The dotted line is an asymptotic value of the curvature scalar as τ goes to infinity. Note that the comoving time is defined by $\tau > 0$. The curvature scalar comes to be a negative constant around $\tau = 0$ and a positive constant as τ goes to infinity. This fact indicates that there is the phase transition from anti-de Sitter universe to de Sitter universe. This figure is plotted in the case of $\alpha = 1$, $\beta = 1$, $A = -2$, $B = 1$, $\kappa = 1$, and $\theta = 4$ in this BPP model.

Now, the solutions (33) and (34) should be satisfied with the constraint (12), which determines the integration function $\kappa t_\pm$,

$$\kappa t_\pm = \frac{1}{4}\kappa\theta^2 \left[\kappa(\alpha e^{-\kappa\theta t} - \beta e^{\kappa\theta t}) - 4\alpha\beta\right]. \tag{39}$$

Then, the induced energy-momentum tensors (14), and (15) are obtained as

$$T^{\rm qt}_{\pm\pm} = -\frac{1}{4}\kappa\theta^2 \left[\kappa(\alpha e^{-\kappa\theta t} + \beta e^{\kappa\theta t}) - 4\beta e^{\kappa\theta t}(\alpha e^{-\kappa\theta t} - \beta e^{\kappa\theta t})\right], \tag{40}$$

$$T^{\rm qt}_{+-} = \frac{1}{2}\beta\kappa^2\theta^2 e^{\kappa\theta t}. \tag{41}$$

Using Eqs. (17) and (18), the energy density, the pressure are explicitly given as

$$\varepsilon = -\frac{1}{2}\kappa^2\theta^2 e^{2\kappa\theta t}(\alpha e^{-\kappa\theta t} + \beta e^{\kappa\theta t} + A)(\alpha e^{-\kappa\theta t} - \beta e^{\kappa\theta t})\left(1 - \frac{4}{\kappa}\beta e^{\kappa\theta t}\right), \tag{42}$$

$$\begin{aligned} p = & -\frac{1}{2}\kappa^2\theta^2 e^{2\kappa\theta t}(\alpha e^{-\kappa\theta t} + \beta e^{\kappa\theta t} + A)\Big[\alpha e^{-\kappa\theta t} + 3\beta e^{\kappa\theta t} \\ & -\frac{4}{\kappa} e^{\kappa\theta t}(\alpha e^{-\kappa\theta t} - \beta e^{\kappa\theta t})\Big], \end{aligned} \tag{43}$$

so that the state parameter $\omega(\tau(t))$ reads

$$\omega = \frac{\alpha e^{-\kappa\theta t} + 3\beta e^{\kappa\theta t} - \frac{4}{\kappa}e^{\kappa\theta t}(\alpha e^{-\kappa\theta t} - \beta e^{\kappa\theta t})}{(\alpha e^{-\kappa\theta t} - \beta e^{\kappa\theta t})\left(1 - \frac{4}{\kappa}\beta e^{\kappa\theta t}\right)} \tag{44}$$

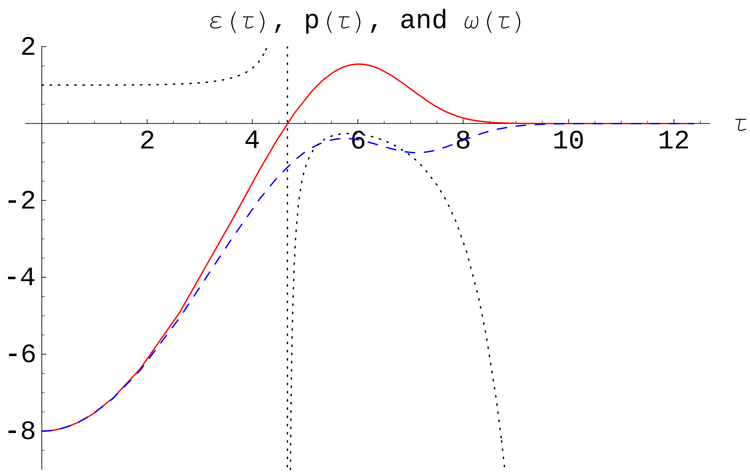

FIG. 2: The solid, the dashed, and the dotted lines denote the energy density, the pressure, and the state parameter of perfect fluid. Note that the pressure is always negative, so that the state parameter can be exotic. This figure is plotted with the same constants used in Fig. 1.

where its profile is plotted in Fig. 2 for the special case giving the AdS-DS transition. The energy density and the pressure are the same value of $-1/2\alpha^2\kappa^2\theta^2$ approximately at the initial time $\tau = 0$ corresponding to $t \to -\infty$, and then the state parameter becomes $\omega \approx 1$. The energy density becomes zero at the comoving time $\tau = \tau_2$, where $\tau_2 = \int_{-\infty}^{t_2} e^{\rho(t)} dt$ where $t_2 \equiv [1/(\kappa\theta)] \ln[\kappa/(4\beta)]$. It changes from negative value to positive around $\tau = \tau_2$, but the pressure is always negative. The state parameter diverges at τ_2 since the energy density vanishes faster than the pressure. The decelerated expansion of the early universe is due to the negative energy density with the negative pressure induced by quantum back reaction ($\omega > 0$), and the accelerated late-time universe comes from the positive energy and the negative pressure which behave like dark energy source ($\omega < 0$).

V. DISCUSSION

We have shown that the phase changing transition from the AdS to the DS phase is possible by assuming the modified Poisson brackets to the semiclassical equations of motion in the BPP model. The usual BPP model does not generate this kind of transition since the integration function $t_\pm$ related to the vacuum state is trivially constant, and the equation of state parameter is simply one which is independent of the time. So, we have taken the nontrivial Poisson brackets at the semiclassical level to overcome this triviality.

The modified Poisson brackets are not the quantum commutators so that it does not mean the quantization of the quantization since the fields Ω and χ are not the operators. In fact, the modified Poisson brackets can be applied to any stage of quantization in order to modify the original equations of motion. For example, if one considers the modified Poisson brackets at the classical dilaton gravity, then the corresponding solution can be obtained, however, it is difficult to obtain the meaningful solution in spite of its complexity. The other heuristic example may be a two-dimensional simple harmonic oscillator with the mass m and the spring constant k, where its Hamiltonian is like $H = (p_x^2 + p_y^2)/(2m) + k(x^2+y^2)/2$. The conventional Poisson brackets generate the two independent set of Hamiltonian equations of motion and then the well-known harmonic solutions are obtained. On the other hand, at this classical level, if we assume the modified Poisson brackets, $\{x, p_x\}_{\rm MPB} = \{y, p_y\}_{\rm MPB} = 1$, $\{p_x, p_y\}_{\rm MPB} = \theta$, then the Hamiltonian equations of motion are modified and the equations of motion can be written in the second order form of $\ddot{x} + \omega^2 x = 2a\dot{y}$, $\ddot{y} + \omega^2 y = -2a\dot{x}$, where $\omega = \sqrt{k/m}$ and $a = \theta/(2m)$. The first order of Hamiltonian equations of motion have been written in the form of the second order Euler-Lagrange equations of motion in order to show the explicit difference between the noncommutative case and the commutative case. Then, the solutions are $x = x_0 \cos at \cos(\omega' t + \varphi_1) + y_0 \sin at \cos(\omega' t + \varphi_2)$, $y = y_0 \cos at \cos(\omega' t + \varphi_2) - x_0 \sin at \cos(\omega' t + \varphi_1)$, where $\omega' \equiv \sqrt{\omega^2 + a^2}$ and x_0, y_0, φ_1, and φ_2 are constants of integration. Note that these are just modified classical solutions rather than the quantum-mechanical ones.

The equation of state parameter is singular at a certain time as seen in Fig. 2. In order for the phase transition from the ADS ($\omega > 0$) to the DS universe ($\omega < 0$), the state parameter also changes its signature at a certain time, in our case at $\tau = \tau_2$. In fact, there are two options satisfying this condition. If the energy density is always positive then the pressure should change its sign, however, in this model, the pressure is always negative, so that the energy density should change its sign. The latter case gives the singular behavior. Of course, the quantum-mechanically induced energy density allows the negative value.

One might wonder how to derive the nontrivial Poisson brackets which are similar to the noncommutativity in string theory [19]. In the string theory, the noncommutative brackets between the coordinates are derived in the D-brane system applied in the constant external tensor field. This is a higher dimensional realization of the slowly moving point particle on the constant magnetic field. All of these systems can be interpreted as constraint systems

[17], so we can expect our model may be a similar constraint system, however, it remains unsolved.

Acknowledgments

This work was supported by the Science Research Center Program of the Korea Science and Engineering Foundation through the Center for Quantum Spacetime **(CQUeST)** of Sogang University with grant number R11 - 2005 - 021.

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